TABLE VI. Impurity atom analyses of samples used in second-order elastic constant measurements.

	C_{44}[a]	C_S'[a]	Ta[b]	C	O	N	H	Other
Ref. 28	0.2809	0.57	500	50	50	50	···	Other metals present at or slightly above spectroscopic detection limits.
Ref. 25								
low Ta	0.2821	0.571	1175	<20	19	<5	<6	Hf, <80; Ti, Mo, <50 ea.
high Ta	0.2825	0.570	130	<20	19	<5	<6	Ni, Sn, Pb, Zr, Be, V, Cr, Mn, Fe, Co, <15 each.
Present	0.2840	0.5661	100	8	23	4	0.4	P, <30; W, 6
Ref. 24	0.2873	0.5604	1500	···	160	110	26	Zr, 100; Fe, 60
Ref. 27	0.2930	0.5345	(No analysis available)					

[a] In units of 10^{12} dyn/cm^2. [b] In ppm by weight. Ref. 28 does not specify whether by weight or by atom.

seen by comparing the values of the slopes calculated from the TOEC in the last column with the experimental values. The uncertainty limits shown were estimated in the same manner as for the single-crystal data.

IV. DISCUSSION

A. Second-Order Elastic Constants

The second-order elastic constants of Refs. 25 and 28 shown in Table II were measured using the resonant bar method where the elastic compliances S_{ij} are determined directly. These were then used to calculate the elastic constants listed by matrix inversion with a resultant loss in accuracy of C_{11} and C_{12}. However, since $C_{44}=1/S_{44}$ and $C_S'=\frac{1}{2}(S_{11}-S_{12})$, these elastic constants are accurate enough for comparison with those determined directly by ultrasonic methods by the other investigators.

There seems to be a trend in the values of the second-order shear constants C_{44} and C_S' for columbium. For each set of data if C_{44} is higher than average, C_S' is lower. This follows even for the sample of Ref. 27 which is quoted as having some observable porosity and which has a measured density about 0.2% lower than that of the other samples. Yet its value of C_{44} is the largest of any reported. Also, the values of the constant C_L' and the bulk modulus, K, for this sample are larger than for the other samples which is surprising considering its porosity. Even if this sample is omitted from the comparison the trend still exists, the differences between samples, about 2% for both shear constants, being larger than the reported error limits. This does not appear to be an impurity effect as can be seen by comparing the impurity levels of the different samples as listed in Table VI.

The "best" value second-order elastic constants shown in Table II were used in calculating the TOEC. It can be shown that if the quoted uncertainties in these values are correct, they can be neglected in estimating the uncertainties in the TOEC values as was done in the present study. That this assumption is valid is supported by the very good agreement between the values for the two samples seen in Table II, and also by the internal consistency of the data for sample 2. For this sample there is a redundancy in the data so that the constant C_{12} can be calculated in two ways, by $C_{12}=C_{11}-C_S'=1.3321\times10^{12}$ and by $C_{12}=2C_L'-C_{11}-2C_{44}=1.3325\times10^{12}$. These values are the same within the accuracy of measurements.

B. Third-Order Elastic Constants

The measured stress derivative slopes for the single-crystal samples in Table III are in several instances outside the range of their estimated limits. For example, m_{10} for sample 1 both before and after irradiation is almost 10% lower than the value for sample 2. However, internal consistency requirements with the rest of the data seem to indicate that the value for sample 2 is more nearly correct. This can be seen by the "best" value of m_{10} in the last column and by the close agreement between the values of C_{111}, C_{112}, and C_{123} determined from the three sets of data, and which depend partially on m_{10} along with m_1, m_4, and m_{16}. This indicates the presence of some unknown sources of error in the individual data as mentioned previously. However, the close agreement between the three sets of TOEC calculated from the data suggest that these errors tend to be smoothed out by conditions of internal consistency and that any systematic errors are relatively small.

The trouble experienced in obtaining the TOEC of polycrystalline columbium indicates the importance of having good polycrystalline samples for these measurements. It is seen that the elongated grain structure of sample A resulted in the measured second-order elastic constants being only about 1% lower than for sample B. However, this grain structure was apparently the cause of the very large differences in the measured stress derivatives of the two samples and the lack of internal consistency of the data for sample A.

One check on the reliability of the hydrostatic pressure measurements at least is to compare the values